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G. Sacerdoti: THE APPLICATION OF LIOVILLE'S THEOREM TO THE MOTION OF CHARGED PARTICLES IN TIME DEPENDENT ELECTROMAGNETIC FIELDS.

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G. Sacerdoti: THE LIOVILLE'S THEOREME APPLIED TO CHARGED PARTICLES MOTION IN PERIODIC IN TIME ELECTROMAGNETIC FIELDS.

As it is known the motion of particles in the phase space x, y, z, p_x^* , p_y^* , p_z^* does not change the initial uniform density $\rho_0 = \frac{dN}{dx\,dy\,dz\,dp_x^*\,dp_y^*\,dp_z^*}$ if the only acting forces are electromagnetic (1)(x).

We divide the phase space in two domains D_1, D_2. In the first D_1 there are the particles which remain at finite (in momentum and in coordinates); in the other D_2 the particles whose trajectories go to infinite.

The shape of D_1 and D_2 versus time is not variable if the electromagnetic field is stationary; otherwise it is in general variable by time.

To "capture" particles emitted from a source

(x) - That is not thrue when there are present radiation losses because in that case there are present also the internal forces of particle that forbid the particles to disintegrate.

these two conditions must be satisfied:

a) the particles have to remain at finite

b) the particles that originally are in the "source" must no anymore pass in the region where the source is. The source is represented as a portion of volume in the domain D_1.

If the volume of D_1 is finite (that is in the most of examples we have in practice) it is easy to demonstrate that capture is no possible also in periodic electromagnetic fields.

We call V_1 the volume of D_1. The total number of particles that remains always in V_1 is $N_1 = \rho_0 V_1$. The volume of the source S be V_S and the particles in the source region at time t_0 be $\rho_0 V_S$.

We consider the distribution of particles at time t_0, $t_0+\tau$, $t_0+2\tau$, ... $t_0+n\tau$ (τ period of electromagnetic fields).

If there were capture the particles at time t_0 in V_S should pass in V_1 after a time $T < k\tau$ has elapsed.

After the time $t_0 + n_0\tau$ if $n_0 > n_1 = \frac{V_1}{V_S} k$, in the volume $V_1 - V_S$ we should have a number of particles greater than $\frac{n_0 \rho_0 V_S}{k} > \rho_0 V_1$ and the new density of particles should have been increased and this does not agree with the Lioville's theoreme.

So it is impossible to have capture by periodic electromagnetic field. A little more complicated demonstration shows that the assumption D_1 of finite volume is not necessary for our conclusion.

Bibliography

(1) - Singe and Griffith: Principles of Mechanics - Mc Graw C. - pag. 480.